Improving hydrostatic performance of 1-3 piezocomposites

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Type 1-3 piezocomposites consist of aligned piezoelectric rods embedded in a passive polymer matrix and have better hydrostatic behavior than single-phase piezoelectric materials. The improved hydrostatic performance of 1 3 piezocomposites comes from two major effects: the axial stress amplification and the lateral stress reduction in the piezoelectric rods. An improved micromechanical model has been developed to predict the hydrostatic response of 1-3 piezocomposites and to examine the interaction between the piezoelectric rods and the polymer matrix. Effective hydrostatic piezoelectric constants for the composite are predicted and compared to those calculated using a generalized plane strain model. The influence of matrix stiffness, interlayer stiffness, rod aspect ratio, and rod volume fraction on the load transfer and the effective hydrostatic piezoelectric coefficient was investigated. The results obtained provide quantitative information on how various parameters affect the hydrostatic response of the composites. © 1995 American Institute of Physics.

I. INTRODUCTION

Lead zirconate titanate (PZT), which is used commonly as an ultrasonic transducer material, suffers from several disadvantages when used as an underwater hydrophone sensor. The hydrostatic response of a PZT ceramic is shown schematically in Fig. 1. The stress-induced electric displacement $D_3$ can be resolved into two parts: One corresponds to the axial stress and the other corresponds to the lateral stress. These two parts are opposite in sign, thus, the net piezoelectric effect is small. The hydrostatic piezoelectric coefficient $d_{33}$ of PZT is low due to the opposite sign of the piezoelectric charge coefficients, $d_{33}$ and $d_{31}$, even though the magnitudes of both $d_{33}$ and $d_{31}$ are large. The hydrostatic voltage coefficient $g_{33} = d_{33}/E_{33}$ is small because of the high dielectric constant $E_{33}$.

An effective way to increase the magnitude of $d_{33}$ and $g_{33}$ is the use of a composite structure design. A variety of piezocomposites can be made by combining a piezoelectric ceramic with a passive polymer. Such composites are classified according to their connectivity. Among these various composites, piezoelectric composites with 1-3 connectivity have attracted a great deal attention and have been used widely for underwater hydrophone applications. The number “1” indicates that the piezoelectric phase is self-connected in one dimension and the number “3” means that the polymer phase is three-dimensionally self-connected throughout the composite. A typical 1-3 piezocomposite, as shown in Fig. 2, consists of an array of aligned, piezoelectric ceramic rods embedded in a passive polymer matrix.

The hydrostatic response of a 1-3 piezocomposite is shown schematically in Fig. 3. The volume average of the stress-induced electric displacement is dependent on the volume average axial stress and volume average lateral stress in the piezoceramic rods. Because of the stress transfer from the piezoelectrically passive polymer phase to the piezoelectrically active ceramic phase, the hydrostatic response of this piezocomposite can be dramatically improved over the single-phase piezoceramic. In designing 1-3 piezocomposites, the primary goal is to maximize the electromechanical coupling. This goal can be achieved either by increasing the axial stress or by effectively reducing the lateral stress in the piezoceramic rods.

Development of an analytical model for predicting the hydrostatic behavior of 1-3 piezoelectric composites is essential in order to understand the stress transfer and its induced phenomena in the composites and to optimize constituent properties for maximum sensitivity. Most of the existing theoretical works concerning the electromechanical coupling phenomena in piezocomposites involve the derivation of relations for the effective elastic constants and effective piezoelectric constants. Classical models such as the Voigt model, which assumes constant strain in the material, and the Reuss model, which assumes constant stress, have been utilized. More refined models include the parallel-serial model and the model by Smith. These simple models are all based on assumptions that either a strain component or the corresponding stress component is constant in the two phases. In addition, Jensen used a concentric cylinder model to characterize 1-3 piezocomposites. This model, which estimates the six technically most important constants of the nine constants in the $e$ set, consists of a piezoelectric rod and a concentric elastic tube subjected to axisymmetric load. Because the displacement assumption introduced in this study is the same as the generalized plane strain solution, the concentric tube model also leads to homogeneous strain along the axial direction of the cylinder. Cao, Zhang, and Cross presented a theoretical study on the static performance of 1-3 piezocomposites. Inhomogeneous displacement profiles were derived for a single-rod composite and a single-tube 1-3 ceramic-polymer composite under uniaxial or hydrostatic stress. The effect of lateral stress is not included in the study and it is essentially a one-dimensional analysis.

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J. Appl. Phys. 77 (9), 1 May 1995

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Recent developments of micromechanics of piezocomposites include the work of Dunn and Taya\(^7\) who estimated the effective properties of a two-phase piezocomposite using dilute, self-consistent, Mori–Tanaka and differential micromechanical models. Progress in micromechanical analysis of piezocomposites has also been made by Benveniste\(^8\)–\(^10\) and Benveniste and Dvorak\(^11\), however, these micromechanics models have not been used extensively to examine the influence of various parameters on the effective piezoelectric constants especially for 1-3 piezocomposites.

The present work is a continuation of an earlier study of displacement and stress fields in 1-3 piezocomposites.\(^12\) The concentric finite composite cylinder model shown in Fig. 4 is chosen as the representative volume element (RVE) for a 1-3 piezocomposite. The length of the composite cylinder model is chosen as the length of the piezoceramic rods embedded in the polymer matrix. As the length of the rods approaches infinity (continuous rods), this composite cylinder model will approach the well-known composite cylinder assemblage of Hashin and Rosen\(^13\) and Hashin.\(^14\) Recently Benveniste\(^15\) has adopted the composite cylinder assemblage model and derived the exact expressions for nine out of ten effective constants which characterize two-phase fibrous piezocomposites.

Because the actual stress transfer is through the interface between the rods and the matrix, the incorporation of any existing interphase region into the micromechanical analysis of 1-3 piezocomposites may be important for understanding composite behavior. A thin interlayer is introduced between the piezoceramic rods and the matrix in the micromechanical model. It is proposed that the presence of this thin interlayer can influence the hydrostatic behavior of the composite and change the sensitivity. Kim, Rittenmyer, and Kahn\(^16\) have already shown that by casting a soft epoxy interlayer between PZT rods and a stiff polymer preform matrix, a 1-1-3 piezocomposite with high hydrostatic charge constant can be made. Sherrit, Wiederick, and Mukherjee\(^17\) also described a 1-3-type PZT-air piezocomposite with extremely high effective hydrostatic piezoelectric constants and figure of merit.

A parametric study is performed to systematically examine the influence of matrix stiffness, rod aspect ratio, interlayer stiffness, and rod volume fraction on the hydrostatic performance of a 1-3 piezocomposite in terms of its effective hydrostatic piezoelectric coefficient \(d_{31}\) and \(g_h\). A comparison of the current solution with the generalized plane strain models is also made. The results obtained can be used to provide useful guidelines for improving the hydrostatic behavior of 1-3 piezocomposites.
II. PROPERTY PREDICTION

A. Constitutive equations

As shown in the schematic diagram of the three-phase composite cylinder in Fig. 4, the region \( r < r_1 \) (ceramic rod), the region \( r_1 < r < r_2 \) (interlayer), and the region \( r_2 < r < r_3 \) (embedding polymer matrix) are denoted by superscripts (1), (2), and (3), respectively. The piezoelectric rod is assumed to be PZT and poled in the axial direction (z direction); thus, the rod is transversely isotropic while the interlayer and the matrix are considered as isotropic and nonpiezoelectric. Hydrostatic pressure \( p \) is applied to the composite cylinder model.

For a transversely isotropic piezoelectric rod the constitutive relations are

\[
\begin{align*}
T_{rr}^{(1)} &= c_{11}^{(1)} e_r^{(1)} + c_{12}^{(1)} e_\theta^{(1)} + c_{13}^{(1)} e_z^{(1)} - e_{31}^{(1)} E_z^{(1)}, \\
T_{\theta\theta}^{(1)} &= c_{12}^{(1)} e_r^{(1)} + c_{11}^{(1)} e_\theta^{(1)} + c_{13}^{(1)} e_z^{(1)} - e_{32}^{(1)} E_z^{(1)}, \\
T_{zz}^{(1)} &= c_{13}^{(1)} e_r^{(1)} + c_{12}^{(1)} e_\theta^{(1)} + c_{33}^{(1)} e_z^{(1)} - e_{33}^{(1)} E_z^{(1)}, \\
T_{rr}^{(2)} &= c_{44}^{(2)} e_r^{(2)} - e_{12}^{(2)} E_r^{(2)}, \\
T_{\theta\theta}^{(2)} &= T_{rr}^{(2)} = 0, \\
D_r^{(1)} &= e_{13}^{(1)} e_r^{(1)} + e_{11}^{(1)} E_r^{(1)}, \\
D_\theta^{(1)} &= 0, \\
D_z^{(1)} &= e_{31}^{(1)} e_r^{(1)} + e_{11}^{(1)} e_\theta^{(1)} + e_{33}^{(1)} e_z^{(1)} + e_{13}^{(1)} E_z^{(1)},
\end{align*}
\]

where elastic, dielectric, and piezoelectric constants are represented by \( c_{ij}^{(1)}, e_{ij}^{(1)}, \) and \( e_{ij}^{(2)}, \) respectively. The constitutive relations for the isotropic, nonpiezoelectric interlayer, and matrix have the form

\[
\begin{align*}
T_{rr}^{(i)} &= c_{11}^{(i)} e_r^{(i)} + c_{12}^{(i)} e_\theta^{(i)} + c_{13}^{(i)} e_z^{(i)}, \\
T_{\theta\theta}^{(i)} &= c_{12}^{(i)} e_r^{(i)} + c_{11}^{(i)} e_\theta^{(i)} + c_{13}^{(i)} e_z^{(i)}, \\
T_{zz}^{(i)} &= c_{13}^{(i)} e_r^{(i)} + c_{12}^{(i)} e_\theta^{(i)} + c_{33}^{(i)} e_z^{(i)}, \\
T_{rr}^{(i)} &= \frac{1}{2} (c_{11}^{(i)} - c_{12}^{(i)}) e_z^{(i)}, \\
i &= 2,3.
\end{align*}
\]

\[ u_z^{(i)} = A^{(i)} r + \frac{B^{(i)}}{r}, \]
\[ w_z^{(i)} = \xi z, \]
\[ T^{(i)}_{rr} = c_{11}^{(i)} (A^{(i)} - \frac{B^{(i)}}{r^2}) + c_{12}^{(i)} (A^{(i)} + \frac{B^{(i)}}{r^2}) + c_{13}^{(i)} \xi - e_{31}^{(i)} E_z^{(i)}, \]
\[ T^{(i)}_{\theta\theta} = c_{12}^{(i)} (A^{(i)} - \frac{B^{(i)}}{r^2}) + c_{11}^{(i)} (A^{(i)} + \frac{B^{(i)}}{r^2}) + c_{13}^{(i)} \xi - e_{32}^{(i)} E_z^{(i)}, \]
\[ T^{(i)}_{zz} = 2 c_{13}^{(i)} (A^{(i)} + \xi) - e_{33}^{(i)} E_z^{(i)}, \]
\[ T^{(i)}_{rz} = T^{(i)}_{\theta z} = T^{(i)}_{r\theta} = E_z^{(i)} = 0, \]

where \( i = 1,2,3, \) and \( A^{(i)}, B^{(i)}, \) and \( \xi \) are constants to be determined by boundary conditions. The boundary conditions for a single embedded ceramic rod with interlayer are as follows:

(1) At \( r = 0, \) the solution is bounded.

(2) The cylinder is subject to homogeneous boundary conditions, i.e.,

\[
\begin{align*}
\text{at } z = \pm l, & \int T_z^{(i)} dr = -p, \\
\text{at } r = r_3, & T_{rr}^{(3)} = -p, \\
E_z^{(1)} &= E_z^{(3)} = E_z^0, \\
E_z^{(1)} - E_z^{(2)} &= E_z^{(2)} - E_z^{(3)} = 0,
\end{align*}
\]

where \( E_z^0 \) is the applied constant electric field. By homogeneous boundary conditions it is meant that when they are applied to a homogeneous solid they result in homogeneous fields.

For both the rod-interphase boundary and the interphase-matrix boundary, continuity of displacements and traction (perfect adhesion) is assumed. Thus, the interface boundary conditions are expressed as

\[
\begin{align*}
\text{at } r = r_1, & u(1) = u(2), & T_{rr}^{(1)} = T_{rr}^{(2)}, \\
\text{at } r = r_2, & u(2) = u(3), & T_{rr}^{(2)} = T_{rr}^{(3)}.
\end{align*}
\]

B. Generalized plane strain models

Generalized plane strain solutions assume the axial strain in the rod, interlayer, and matrix is uniform such that \( e_z^{(i)} = \zeta, \) where \( \zeta \) is a constant to be determined. Axially symmetric displacements in all three constituents are of the form

\[
\begin{align*}
\nu_r^{(i)} &= u_r^{(i)}(r), \\
\nu_\theta^{(i)} &= 0, \\
\nu_z^{(i)} &= \zeta z, \\
i &= 1,2,3.
\end{align*}
\]

Substitution of the constitutive relations and the well-known strain-displacement relations into the equilibrium equations yields the generalized plane strain solution

J. Appl. Phys., Vol. 77, No. 9, 1 May 1995

L. Li and N. R. Sottos 4597
cause of the homogeneous boundary conditions applied. This assumption is a fairly accurate one and has been used in most previous works to predict effective properties.\textsuperscript{4–6} Using the form of the displacements in Eq. (10) and the electric field in Eq. (7), the governing equations are reduced to the following two equations:

\[
\begin{align*}
\alpha_{11}^{(i)} \left[ \frac{\partial^2 u^{(i)}}{\partial x^2} + \frac{1}{r} \frac{\partial u^{(i)}}{\partial r} \right] + (c_{13}^{(i)} + c_{44}^{(i)}) \frac{\partial^2 w^{(i)}}{\partial r \partial z} + c_{44}^{(i)} \frac{\partial^2 u^{(i)}}{\partial z^2} &= 0, \\
\alpha_{44}^{(i)} \left[ \frac{\partial^2 w^{(i)}}{\partial r^2} + \frac{1}{r} \frac{\partial w^{(i)}}{\partial r} \right] + (c_{13}^{(i)} + c_{44}^{(i)}) \frac{\partial^2 u^{(i)}}{\partial r \partial z} + c_{44}^{(i)} \frac{\partial^2 w^{(i)}}{\partial z^2} &= 0,
\end{align*}
\]

where \( i = 1, 2, 3 \), and in the isotropic matrix and interlayer \( c_{11}^{(i)} = c_{13}^{(i)}, c_{12}^{(i)} = c_{33}^{(i)}, \) and \( c_{44}^{(i)} = 0.5(c_{11}^{(i)} - c_{12}^{(i)}) \).

The complementary solution to Eq. (11) for the piezoceramic rod \(( i = 1)\) is achieved by using two displacement potential functions. The displacements are written as

\[
\begin{align*}
u_{c}^{(1)} &= \frac{\partial}{\partial r} (\psi_{1} + \psi_{2}), \\
w_{c}^{(1)} &= \frac{\partial}{\partial z} (s_{1} \psi_{1} + s_{2} \psi_{2}).
\end{align*}
\]

The constants \( s_{j} \) \(( j = 1, 2)\) are determined by

\[
s_{j} = \frac{p_{j} c_{11}^{(i)} - c_{13}^{(i)}}{c_{11}^{(i)} + c_{44}^{(i)}} \quad (j = 1, 2),
\]

where \( p_{j}^{2} \) and \( p_{j}^{2} \) are the roots of the following equation:

\[
c_{11}^{(i)} c_{13}^{(i)} p^{4} + [c_{11}^{(i)}(c_{11}^{(i)} + 2c_{44}^{(i)}) - c_{12}^{(i)} c_{13}^{(i)}] p^{2} + c_{44}^{(i)} c_{13}^{(i)} = 0.
\]

The potential functions \( \psi_{1}(r, z) \) and \( \psi_{2}(r, z) \) must satisfy the displacement equations of equilibrium. Also, the structure of the complementary solution depends on the nature of the value of \( p_{1}^{2} \) and \( p_{2}^{2} \), and hence on the values of the elastic constants. For a PZT-5H rod,\textsuperscript{18} and \( p_{1}^{2} \) and \( p_{2}^{2} \) are complex conjugate. Because the displacements and stresses are real and bounded at \( r = 0 \), the displacement potentials must be complex conjugate functions and are chosen in the form\textsuperscript{12}

\[
\begin{align*}
\psi_{1} &= \sum_{n=1}^{\infty} \cos \mu_{n} z \left[ D_{1n}^{(i)} I_{0}(p_{1} \mu_{n} r) + \frac{D_{3n}^{(i)}}{i} I_{0}(p_{1} \mu_{n} r) \right], \\
\psi_{2} &= \sum_{n=1}^{\infty} \sin \mu_{n} z \left[ D_{1n}^{(i)} I_{0}(p_{1} \mu_{n} r) - \frac{D_{3n}^{(i)}}{i} I_{0}(p_{1} \mu_{n} r) \right],
\end{align*}
\]

where \( I_{0} \) is the zeroth-order modified Bessel function of the first kind and \( i \) represents the pure imaginary number, i.e., \( i^2 = -1 \). \( D_{1n}^{(i)} \) and \( D_{3n}^{(i)} \) are constants to be determined from the boundary conditions. The \( \mu_{n} \) are the eigenvalues determined by the boundary conditions at \( z = \pm l \).

\[\mu_{n} = \frac{n \pi}{2l} \quad (n = 1, 3, 5, \ldots).\]

The complementary displacements and stresses for the ceramic rod \(( i = 1)\) are then obtained as\textsuperscript{12}

\[
\begin{align*}
u_{c}^{(1)} &= \sum_{n=1}^{\infty} \cos(\mu_{n} z) \left[ D_{1n}^{(1)} h_{1n}^{(1)}(r) + D_{3n}^{(1)} h_{2n}^{(1)}(r) \right], \\
w_{c}^{(1)} &= \sum_{n=1}^{\infty} \sin(\mu_{n} z) \left[ D_{1n}^{(1)} h_{3n}^{(1)}(r) + D_{3n}^{(1)} h_{4n}^{(1)}(r) \right], \\
n_{c}^{(1)} &= \sum_{n=1}^{\infty} \cos(\mu_{n} z) \left[ D_{1n}^{(1)} h_{5n}^{(1)}(r) + D_{3n}^{(1)} h_{6n}^{(1)}(r) \right], \\
T_{rr}^{(1)} &= \sum_{n=1}^{\infty} \cos(\mu_{n} z) \left[ D_{1n}^{(1)} h_{7n}^{(1)}(r) + D_{3n}^{(1)} h_{8n}^{(1)}(r) \right], \\
T_{\theta\theta}^{(1)} &= \sum_{n=1}^{\infty} \cos(\mu_{n} z) \left[ D_{1n}^{(1)} h_{9n}^{(1)}(r) + D_{3n}^{(1)} h_{10n}^{(1)}(r) \right], \\
T_{\theta z}^{(1)} &= \sum_{n=1}^{\infty} \cos(\mu_{n} z) \left[ D_{1n}^{(1)} h_{11n}^{(1)}(r) + D_{3n}^{(1)} h_{12n}^{(1)}(r) \right],
\end{align*}
\]

where functions \( h_{1n}^{(1)}(r) - h_{12n}^{(1)}(r) \) are given in Ref. 12 and \( D_{1n}^{(1)} \) and \( D_{3n}^{(1)} \) are sets of constants to be determined from the boundary conditions.

For the isotropic, nonpiezoelectric matrix and interlayer, the complementary solution to Eq. (11) is obtained using Love’s stress function \( f^{(i)}(r, z) \). With this approach, the solution in the matrix and the interlayer takes the following form:\textsuperscript{12}

\[
\begin{align*}
u_{c}^{(i)} &= \sum_{n=1}^{\infty} \cos(\mu_{n} z) \left[ D_{1n}^{(i)} h_{1n}^{(i)}(r) + D_{3n}^{(i)} h_{2n}^{(i)}(r) + D_{2n}^{(i)} s_{1n}^{(i)}(r) \right] \\
&\quad + D_{4n}^{(i)} s_{2n}^{(i)}(r), \\
w_{c}^{(i)} &= \sum_{n=1}^{\infty} \sin(\mu_{n} z) \left[ D_{1n}^{(i)} h_{3n}^{(i)}(r) + D_{3n}^{(i)} h_{4n}^{(i)}(r) + D_{2n}^{(i)} s_{3n}^{(i)}(r) \right] \\
&\quad + D_{4n}^{(i)} s_{4n}^{(i)}(r), \\
n_{c}^{(i)} &= \sum_{n=1}^{\infty} \cos(\mu_{n} z) \left[ D_{1n}^{(i)} h_{5n}^{(i)}(r) + D_{3n}^{(i)} h_{6n}^{(i)}(r) + D_{2n}^{(i)} s_{5n}^{(i)}(r) \right] \\
&\quad + D_{4n}^{(i)} s_{6n}^{(i)}(r),
\end{align*}
\]
\[ T_{zz}^{(i)} c = \sum_{n=1}^{\infty} \cos(\mu_n z)[D_{1n}h^{(i)}_{1n}(r) + D_{3n}h^{(i)}_{3n}(r)] + D_{4n}h^{(i)}_{4n}(r), \]

\[ T_{zz}^{(i)} c = \sum_{n=1}^{\infty} \cos(\mu_n z)[D_{11}h^{(i)}_{11}(r) + D_{33}h^{(i)}_{33}(r)] + D_{43}h^{(i)}_{43}(r), \]

where \( i = 2,3 \), \( h^{(i)}_{11}(r) - h^{(i)}_{22}(r) \) are expressions containing the modified Bessel functions of the first kind, \( I_0(\mu_n r) \) and \( I_1(\mu_n r) \), and \( g^{(i)}_{11}(r) - g^{(i)}_{22}(r) \) are expressions containing modified Bessel functions of the second kind, \( K_0(\mu_n r) \) and \( K_1(\mu_n r) \). The functions \( h^{(i)}_{11}(r) - h^{(i)}_{22}(r) \) and \( g^{(i)}_{11}(r) - g^{(i)}_{22}(r) \) are given in Ref. 12. The constants \( D_{11}^{(i)}, D_{22}^{(i)}, D_{33}^{(i)} \) and \( D_{43}^{(i)} \) are to be determined from the boundary conditions.

The final solution is achieved by superposing the complementary solution and an additional solution corresponding to a generalized plane strain state in order to satisfy all the boundary conditions. The boundary conditions for the single embedded ceramic rod with interlayer shown in Fig. 4 are as follows.

(1) At \( r = 0 \), the solution is bounded.

(2) The outer surfaces of the cylinder are subject to hydrostatic pressure \( p \) and the shear stresses constitute a system in equilibrium, so that

\[ T_{zz}^{(i)} = -p, \quad T_{rr}^{(i)} d r = 0, \]

at \( z = \pm l, \quad z = \pm \frac{l}{2}, \quad r = r_3 \).

The effective dielectric constant \( \tilde{\varepsilon}_{33} \) of the composite can also be predicted by using the composite cylinder model. Since the dielectric constant of the polymer used here is very small compared with that of the piezoceramic, the effective dielectric constant can be written as

\[ \tilde{\varepsilon}_{33} = \varepsilon_{33}^{(1)} \frac{\varepsilon_{33}^{(1)} v_f}{\varepsilon_{33}^{(1)} v_f}, \]

where \( \varepsilon_{33}^{(1)} \) is the dielectric constant of the piezoceramic and \( v_f \) is the volume fraction of the piezoceramic rods. The effective piezoelectric voltage constant is given by

\[ \tilde{d}_h = \frac{\langle D_{zz}^{(1)} \rangle - \langle \varepsilon_{33}^{(1)} E_z^{(1)} \rangle}{-p}. \]

The figure of merit of the 1-3 piezocomposite defined as \( \tilde{d}_h \tilde{\varepsilon}_{33} \) can be written as follows:

\[ \tilde{d}_h \tilde{\varepsilon}_{33} = \frac{\langle D_{zz}^{(1)} \rangle}{\varepsilon_{33}^{(1)} v_f}. \]

<table>
<thead>
<tr>
<th>Property</th>
<th>PZT-SH</th>
<th>Property</th>
<th>PZT-SH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{11} ) (GPa)</td>
<td>126</td>
<td>( \varepsilon_{33}^{(1)} ) (C/m(^2))</td>
<td>-6.55</td>
</tr>
<tr>
<td>( c_{11} ) (GPa)</td>
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<td>23.3</td>
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<td>( c_{11} ) (GPa)</td>
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<td>( d_{33}^{(1)} (10^{-12} \text{mV}) )</td>
<td>-274</td>
</tr>
<tr>
<td>( c_{11} ) (GPa)</td>
<td>23.0</td>
<td>( d_{33}^{(1)} (10^{-12} \text{mV}) )</td>
<td>593</td>
</tr>
<tr>
<td>( \varepsilon_{33}^{(1)} )</td>
<td>3400</td>
<td>( d_{33}^{(1)} (10^{-12} \text{mV}) )</td>
<td>45</td>
</tr>
</tbody>
</table>

where \( \langle \rangle \) represents volume average. For a homogeneous PZT rod with a hydrostatic piezoelectric constant \( d_h \), the volume average of the pure hydrostatic pressure induced electric displacement is

\[ D_z = d_h (-p) + \varepsilon_{33}^{(1)} E_z^{(1)}. \]

TABLE II. Elastic properties of Spurr epoxy

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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</thead>
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<td>( v )</td>
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</tr>
<tr>
<td>( \nu_0 ) (GPa)</td>
<td>2.101</td>
</tr>
</tbody>
</table>
III. IMPROVING THE HYDROSTATIC PERFORMANCE THROUGH AXIAL STRESS AMPLIFICATION

A. $\tilde{d}_h$, $\tilde{g}_h$ predicted and the axial stress amplification in the PZT rods

Predictions of the effective hydrostatic piezoelectric constant $\tilde{d}_h$ for a typical PZT-5H rod aspect ratio ($2l/d=13.3$) using the finite concentric cylinder model are plotted in Fig. 5 for a range of rod volume fractions (no interlayer). Here $2l$ is the length of the PZT rods and $d$ is the diameter. The effective $\tilde{d}_h$ exceeds the $d_h$ of the constituent ceramic, approaching three times as high as that of the pure PZT-5H ($d_h=4.5\times10^{-12}$ V m/N). The prediction of $\tilde{d}_h$ made from the generalized plane strain model is also shown in Fig. 5. The value corresponding to the generalized plane strain model is larger than that of the finite concentric cylinder model. Additionally, the peak value of $\tilde{d}_h$ shifts to a lower-volume fraction region for the plane strain prediction.

Calculated values of $\tilde{g}_h$ and $\tilde{d}_h\tilde{g}_h$ are plotted in Figs. 6 and 7, respectively as a function of the rod volume fraction ($2l/d=13.3$). The $\tilde{g}_h$ peaks at low ceramic content, attaining a value greater than that for the pure ceramic ($g_h=1.5\times10^{-5}$ V m/N). This enhancement is due to the dilution of the dielectric permittivity and the increase of $\tilde{d}_h$.

A somewhat broader peak in the figure of merit, $d_hg_h$, also appears at low PZT-5H content for essentially the same reason as its $\tilde{g}_h$ peak. The value attained is significantly higher than the pure ceramic ($d_hg_h=67.5\times10^{-12}$ V$^2$/m/N). Again, the plane strain model predicts significantly higher values of $\tilde{d}_h$ and $d_h\tilde{g}_h$ than the composite cylinder model.

The volume average stresses in the PZT rods are plotted in Fig. 8 along with the $\tilde{d}_h$ for the same case as in Fig. 5. There is a large stress amplification in the axial stress $\tau_{zz}$ especially for the low PZT rods volume fraction region. Although the lateral stress is also slightly increased, competition among the axial stress, the lateral stress and the volume fraction of the PZT rod gives an enhanced effective hydrostatic piezoelectric constant $\tilde{d}_h$. Thus, axial stress amplification is beneficial for the enhancement of $\tilde{d}_h$.

B. Influence of PZT rod aspect ratio on $\tilde{d}_h$

The aspect ratio of the PZT rods, $2l/d$, has been proven in experiments to be a critical parameter in 1-3 piezocomposites. The effect of the aspect ratio is not included in the simple generalized plane strain models; however, this effect can be studied by using the current finite concentric cylinder model. A parametric study was carried out to assess the influence of PZT-5H rod aspect ratio on $\tilde{d}_h$. Material properties of PZT-5H and the polymer matrix are the same as in Tables I and II. The results are plotted in Fig. 9 for several different aspect ratios. For each case, the $\tilde{d}_h$ exhibits a peak value which shifts to higher volume fractions with decreasing aspect ratio. The peak value increases with increasing aspect ratio, but there is a saturation of the effect for $2l/d > 500$. Thus, the enhancement of axial stress in the PZT rod from increasing the rod aspect ratio has an upper limit. This saturated value of $\tilde{d}_h$ approaches the value predicted by the...
generalized plane strain model. For a composite with small rod aspect ratio, the simple generalized plane strain models significantly over predict the $\tilde{d}_h$. In practical applications, the PZT rod aspect ratios are typically small ($2l/d < 50$) due to poling limitations, and the finite concentric cylinder model would more accurately predict the behavior.

C. Influence of matrix stiffness on $\tilde{d}_h$

The influence of matrix stiffness on $\tilde{d}_h$ of 1-3 piezocomposites was examined by varying the matrix stiffness $(2l/d = 13.3)$ and using the finite concentric cylinder model. The results are plotted in Fig. 10 where $Y^{(3)}$ is the Young’s modulus of the matrix and $Y^0$ is the modulus of plain spurr epoxy listed in Table II. The $\tilde{d}_h$ increases with decreasing matrix stiffness but saturates for a very soft matrix. The effect of softening the matrix is similar to the effect of increasing the PZT rod aspect ratio due to axial stress amplification which is significant at low PZT rod volume fractions.

IV. IMPROVING THE HYDROSTATIC PERFORMANCE THROUGH LATERAL STRESS REDUCTION

In designing 1-3 piezocomposites, the primary goal is to maximize the electromechanical coupling. The enhancement of electromechanical coupling under hydrostatic pressure is achieved either by increasing the axial stress or by decreasing the lateral stress in the piezoceramic rods. The aforementioned results of the influence of the rod aspect ratio and the matrix stiffness on $\tilde{d}_h$ demonstrates how axial stress amplification in the rods can be achieved from these two effects. The results also show the saturation from these two effects.

A. Lateral stress reduction and the role of interlayer

The influence of an interlayer with a thickness of 0.1 mm is shown in Fig. 11, where the $Y^{(2)}$ is the interlayer modulus and $Y^0$ is the modulus of plain spurr epoxy as given in Table II. The PZT-5H rod aspect ratio is fixed at 13.3. The introduction of a thin soft interlayer greatly reduces the lateral stress in the PZT rod. The effect of softening the interlayer becomes dominant for the rod volume fractions between 20% and 45% where the $\tilde{d}_h$ increases significantly with a decrease in the interlayer stiffness.

The effect of the lateral stress reduction for the case of a soft interlayer is illustrated in Fig. 12. The volume average stresses in the PZT rods are plotted in Fig. 12 along with the $\tilde{d}_h$ for $Y^{(2)} = Y^0/100$. A comparison with Fig. 8 reveals that there is a large reduction in the lateral stress for the compliant interlayer. Although the axial stress is also slightly decreased, competition among the axial stress, the lateral stress, and the volume fraction of the PZT rod give an enhanced effective hydrostatic piezoelectric constant $\tilde{d}_h$. Thus, lateral stress reduction through the introduction of a thin, soft interlayer is also beneficial for the enhancement of $\tilde{d}_h$.

B. Influence of the interlayer Poisson ratio on $\tilde{d}_h$

The Poisson ratio of the polymer matrix is also a very important parameter in designing 1-3 piezocomposites for hydrostatic applications. If the Poisson ratio of the polymer

FIG. 8. Variation of $\tilde{d}_h$ and the volume average axial and lateral stresses with volume fraction of PZT rods for the case of no interlayer.

FIG. 9. Variation of $\tilde{d}_h$ with volume fraction of PZT rods for different rod aspect ratio.

FIG. 10. Variation of $\tilde{d}_h$ with volume fraction of PZT rods for different matrix modulus.

J. Appl. Phvs., Vol. 77, No. 9. 1 May 1995
L. Li and N. R. Sottos 4601
matrix is large, the polymer matrix will be hydrostatically incompressible, and large lateral stresses develop in the piezoceramic rods. Previous solutions to suppress the Poisson effect of the polymer matrix relied on introducing bubbles into the polymer matrix to reduce the Poisson ratio; this also reduces the matrix stiffness and introduces an undesired bias pressure dependence to the hydrostatic sensitivity.\textsuperscript{2,19}

The lateral stress in the piezoceramic rods can also be reduced by controlling the Poisson ratio of a thin interlayer. This effect is shown in Fig. 13 for the case of $\gamma^{(2)} = \gamma^0/10$. The thickness of the interlayer is fixed at 0.1 mm and the PZT-5H rod aspect ratio is 13.3. Figure 13 shows a compliant interlayer with a smaller Poisson ratio can further reduce the lateral stress in the PZT rod and increases the $\bar{d}_h$ of the 1-3 piezocomposite.

V. SUMMARY AND CONCLUSIONS

A micromechanics model was developed for studying the hydrostatic response of 1-3 piezocomposites. This solution was then used to predict the effective hydrostatic piezoelectric constants $d_h$, $e_h$, and the figure of merit $\bar{d}_h \bar{e}_h$. Parametric studies were performed to assess the influence of matrix stiffness, rod aspect ratio, interlayer stiffness, and rod volume fraction on the hydrostatic piezoelectric constants. Increasing the aspect ratio significantly increases the $d_h$ due to the axial stress amplification and increased load transfer. This effect saturates for $2L/d>500$ when the solution approaches the values predicted by a generalized plane strain model (infinite aspect ratio). Even though a large aspect ratio would increase hydrostatic sensitivity, piezocomposites are typically fabricated with small rod aspect ratios due to poling limitations. The properties of these piezocomposites are more accurately modeled by a finite concentric cylinder model.

The $d_h$ can also be increased by decreasing the matrix stiffness or introducing a compliant interlayer around the PZT rods. Decreasing the matrix stiffness causes axial stress amplification which leads to higher $d_h$ values; however, difficulties may exist in fabricating an entire composite with a very compliant matrix. A compliant interlayer significantly increases the $d_h$ by effectively attenuating the lateral stress in the PZT rods without decreasing the overall stiffness of the composite. A compliant interlayer with smaller Poisson ratio can further increases the $\bar{d}_h$ of the 1-3 piezocomposite.

Currently, reduction in lateral stress and axial stress amplification are not obtained simultaneously. In order to improve the hydrostatic performance of a 1-3 piezocomposite further, the interlayer must be tailored to enhance both these
effects. An interlayer that was functionally graded along the axis of the rod, such that it was stiffer at the edges of the rod to enhance load transfer and axial stress amplification and softer along the interior region of the rod to reduce lateral stresses, may be able to exploit both effects.

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